Glauber Model + Particle Production Model

Modeling the experimental observables to determine centrality.

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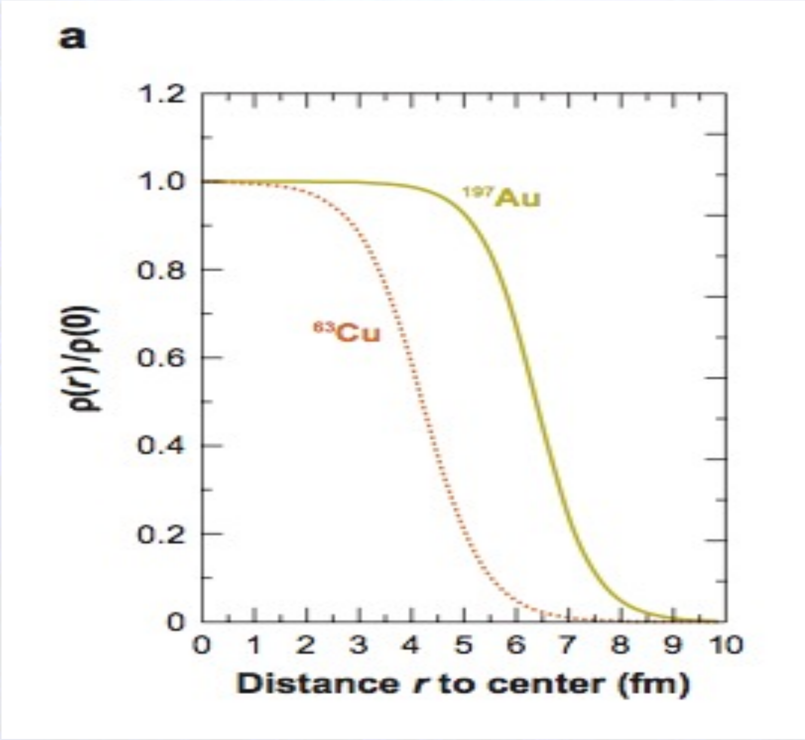
Nuclear Charge Densities

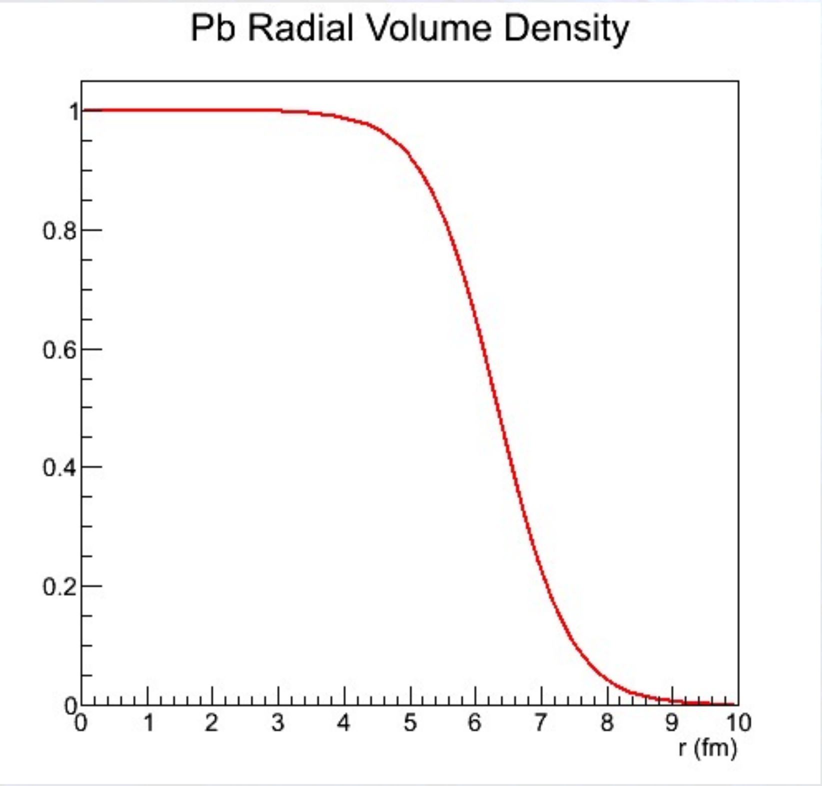
- Charge densities: similar to a hard sphere.
  - Edge is "fuzzy": Woods-Saxon distribution

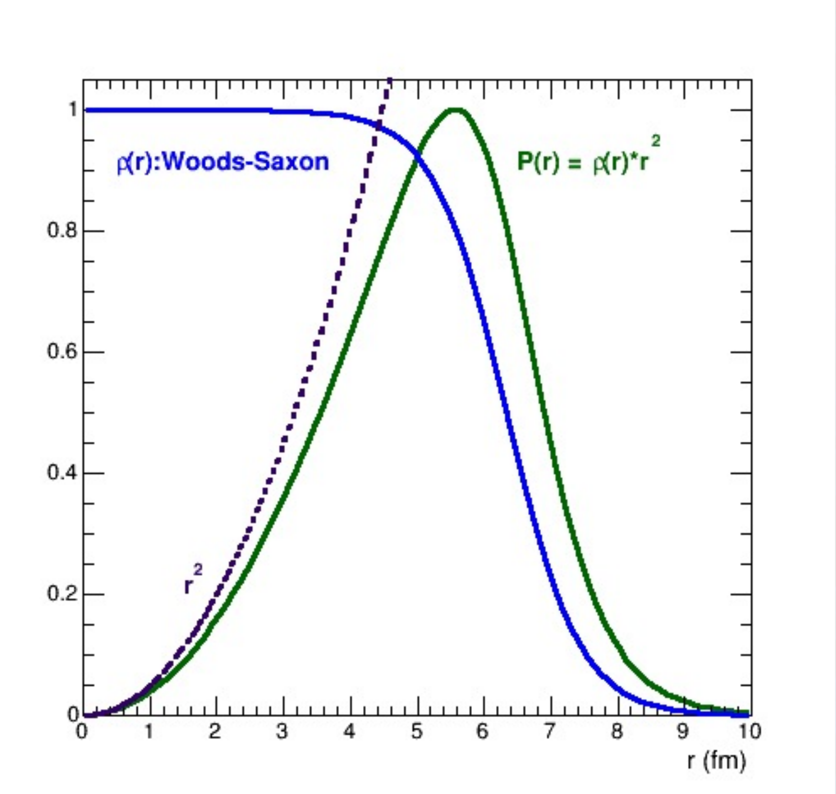


For the Pb nucleus (used at LHC)

- Woods-Saxon density:
  - R = 1.07 fm * A $^{1/3}$
  - a =0.54 fm
  - A = 208 nucleons
- Probability : $\propto r^2 \rho(r)$

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{a}}}$$



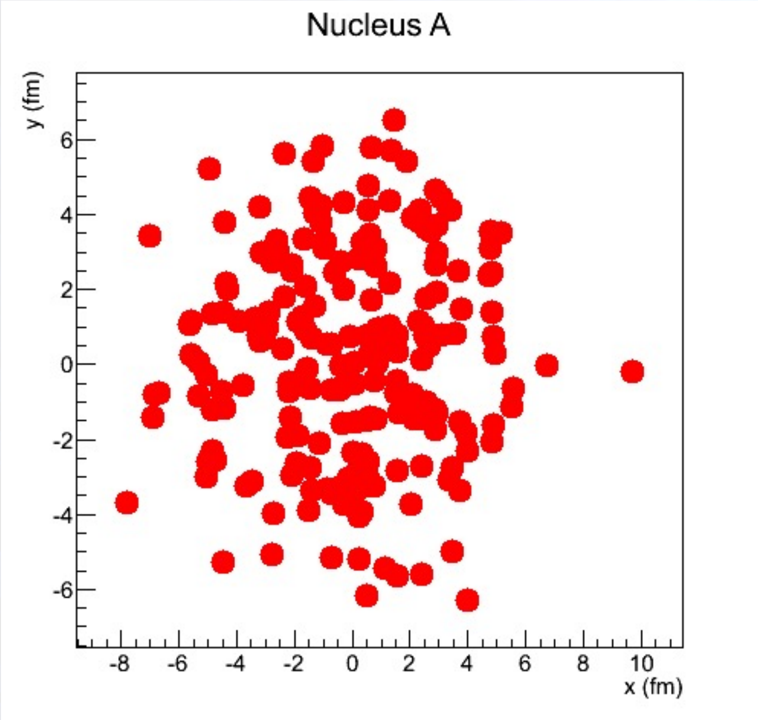


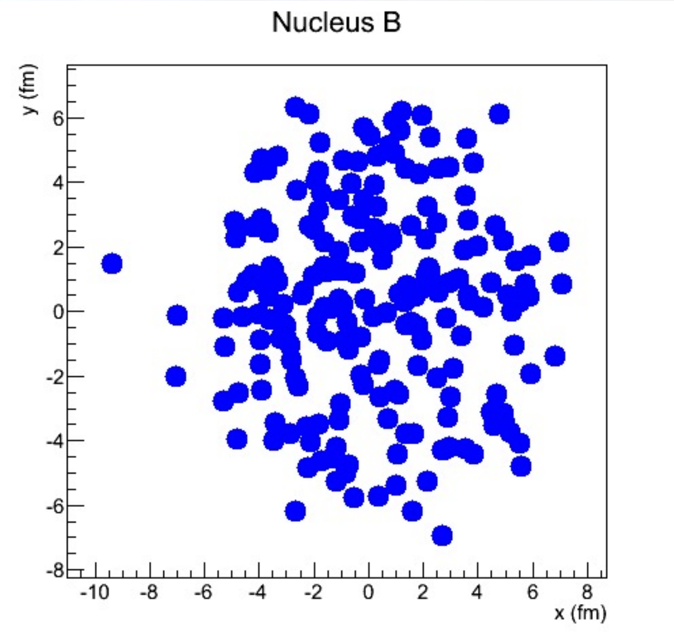
Nuclei: A bunch of nucleons

- Each nucleon is distributed with:

$$P(r,\theta,\phi) = \rho(r)dV = \rho(r)r^2 dr d(\cos\theta) d\phi$$

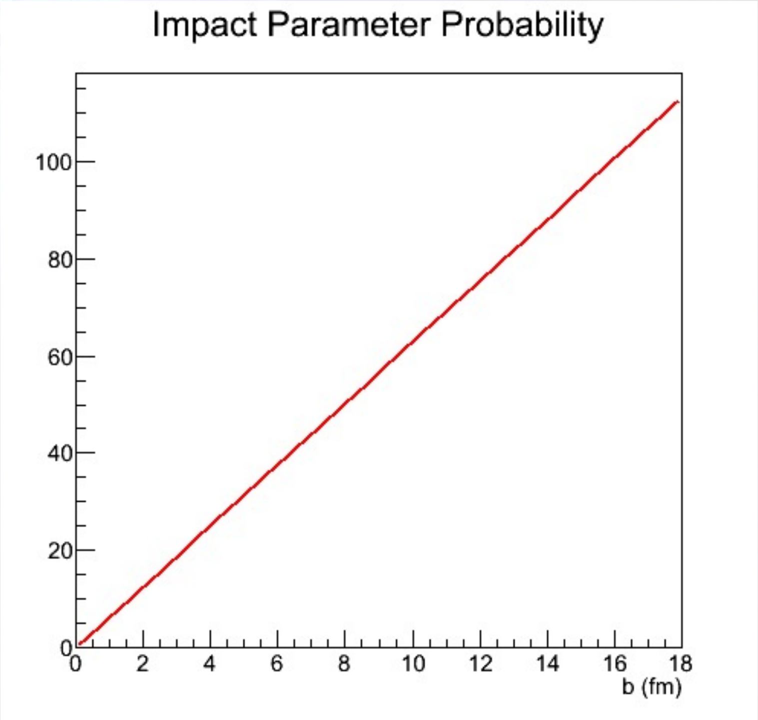
  - Angular probabilities:
    - Flat in ϕ, flat in $\cos(\theta)$.





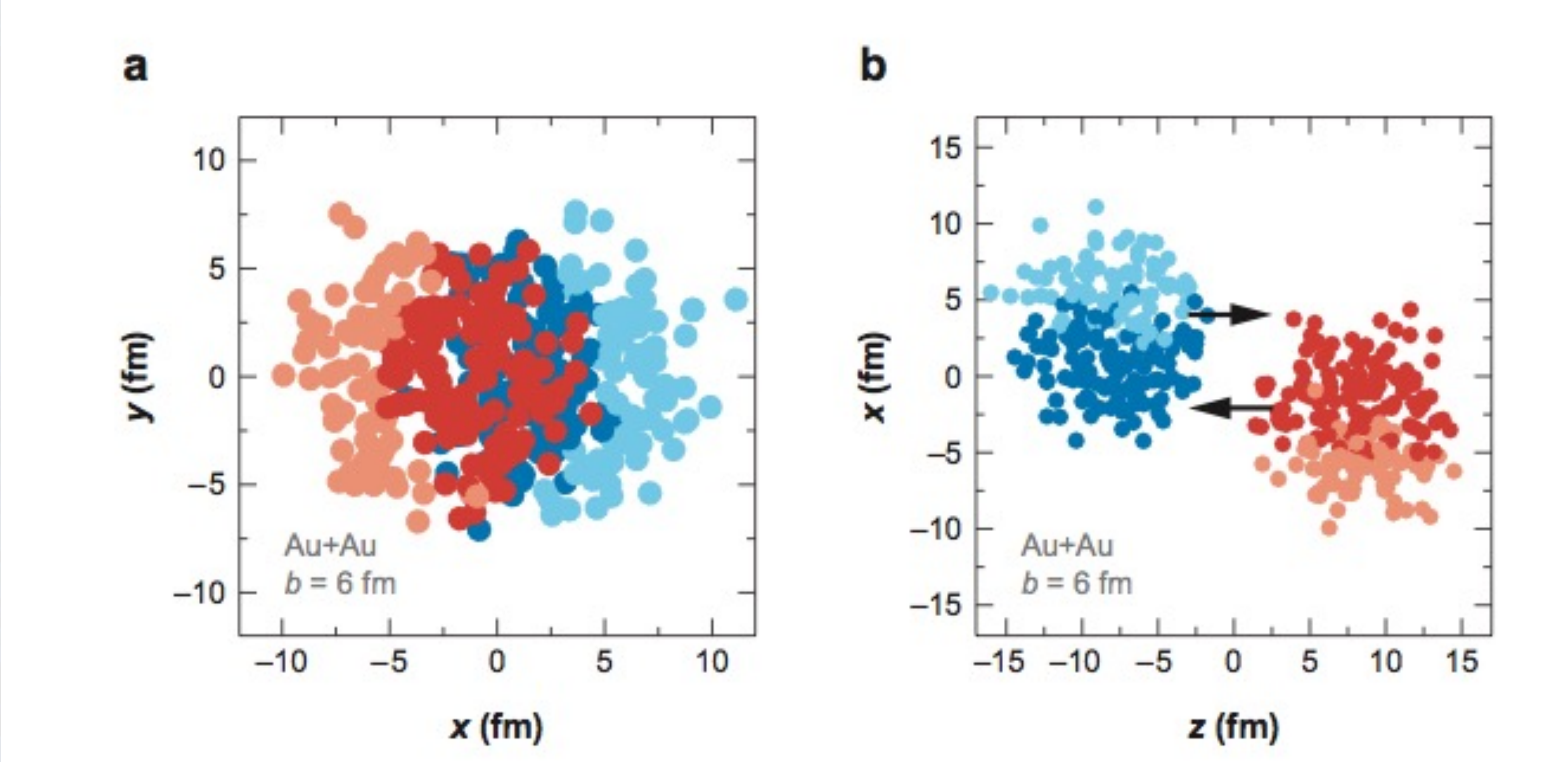
Impact parameter distribution

- Like hitting a target:
- Rings have more area
- Area of ring of radius *b*, thickness *db*: $2\pi b db$
- **Area proportional to probability**



Collision:

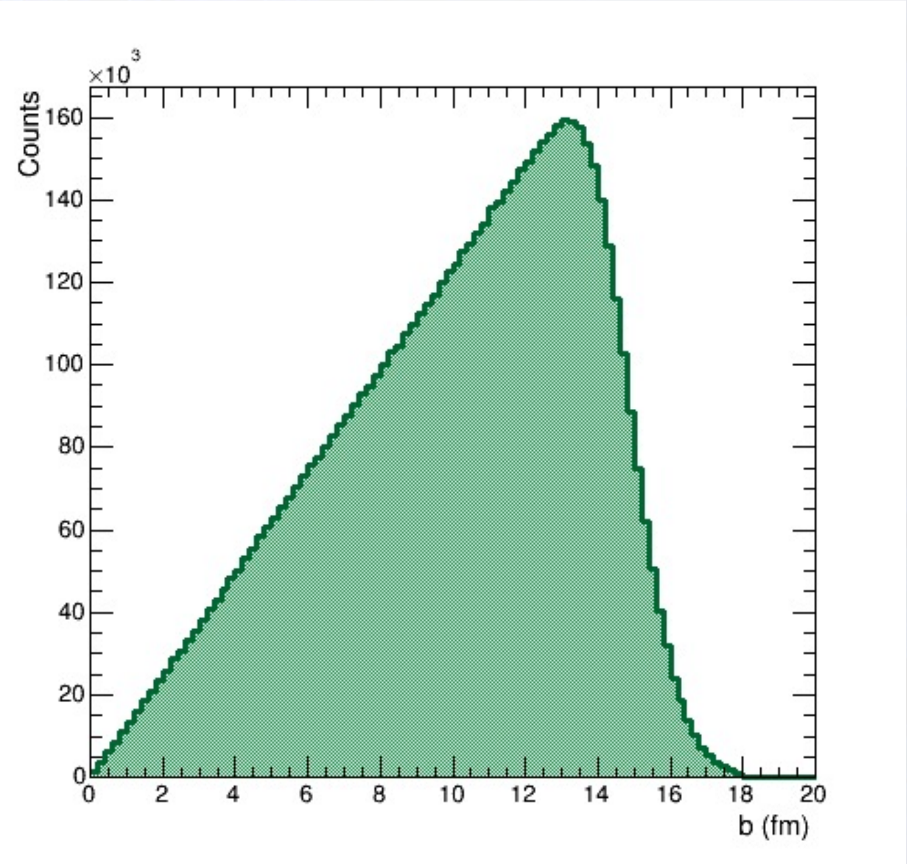
- 2 Nuclei colliding
- Red: nucleons from nucleus A
- Blue: nucleons from nucleus B



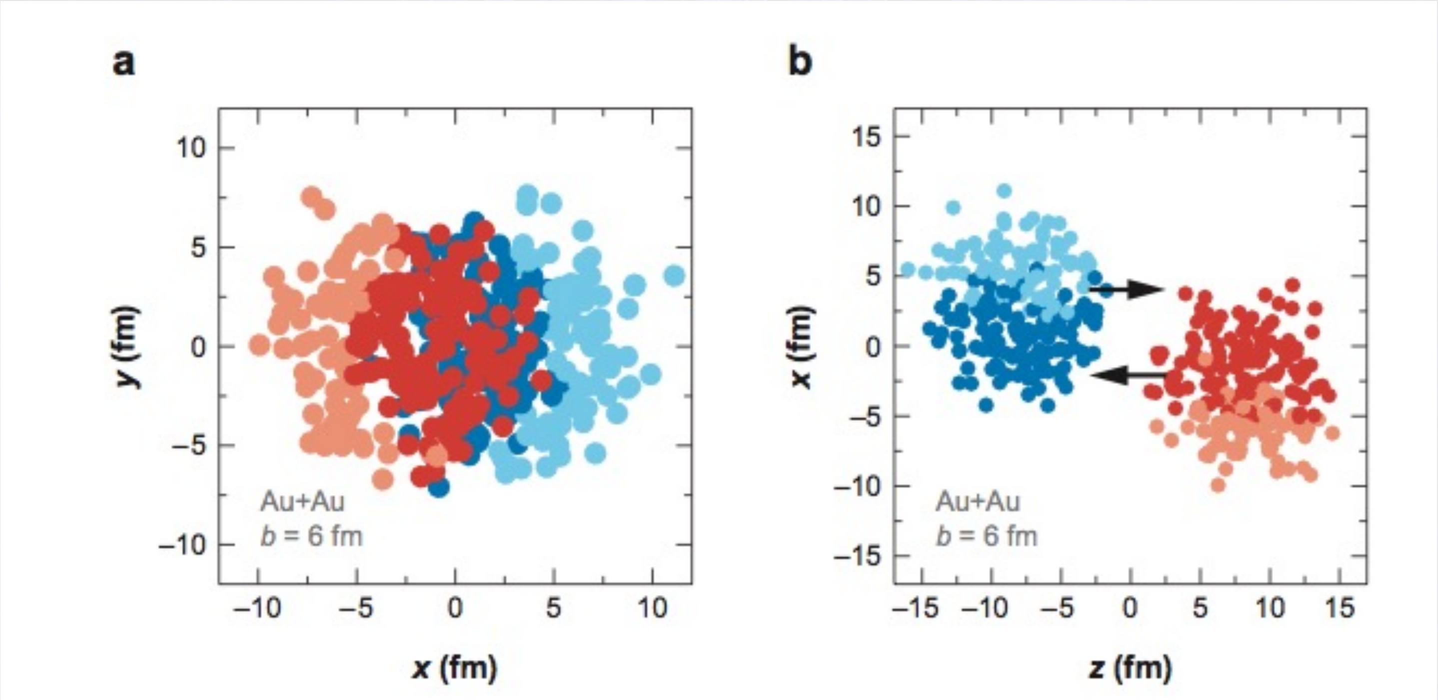
M.L.Miller, et al. Annu. Rev. Nucl. Part. Sci. 2007.57:205-243

Interaction Probability vs. Impact Parameter, b

- After 10M events
- Beyond *b*~2*R* Nuclei miss each other
  - Note fuzzy edge
- Largest probability:
  - Collision at b~12-14 fm
- Head on collisions:
  - b~0: Small probability



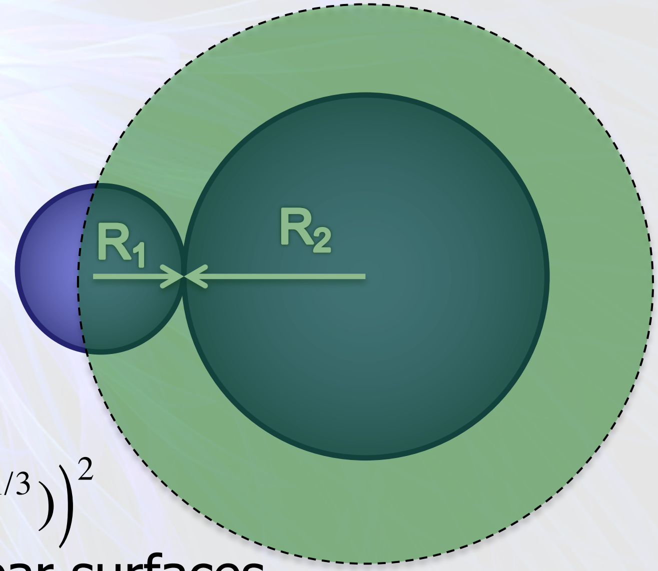
Binary Collisions, Number of participants



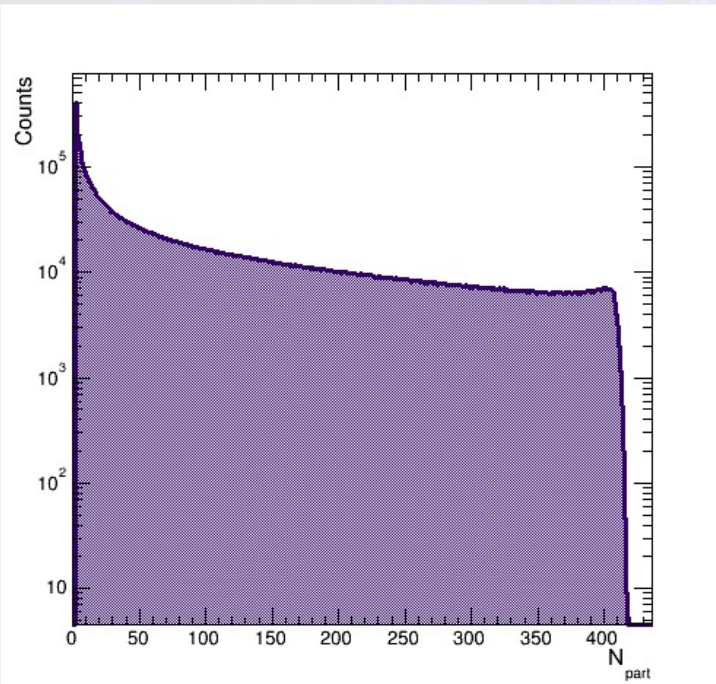
- If two nucleons get closer than $d^2 < \sigma/\pi$ **they collide.**
- **Each colliding nucleon is a "participant" (Dark colors)**
- **Count number of binary collisions.**
- **Count number of participants**

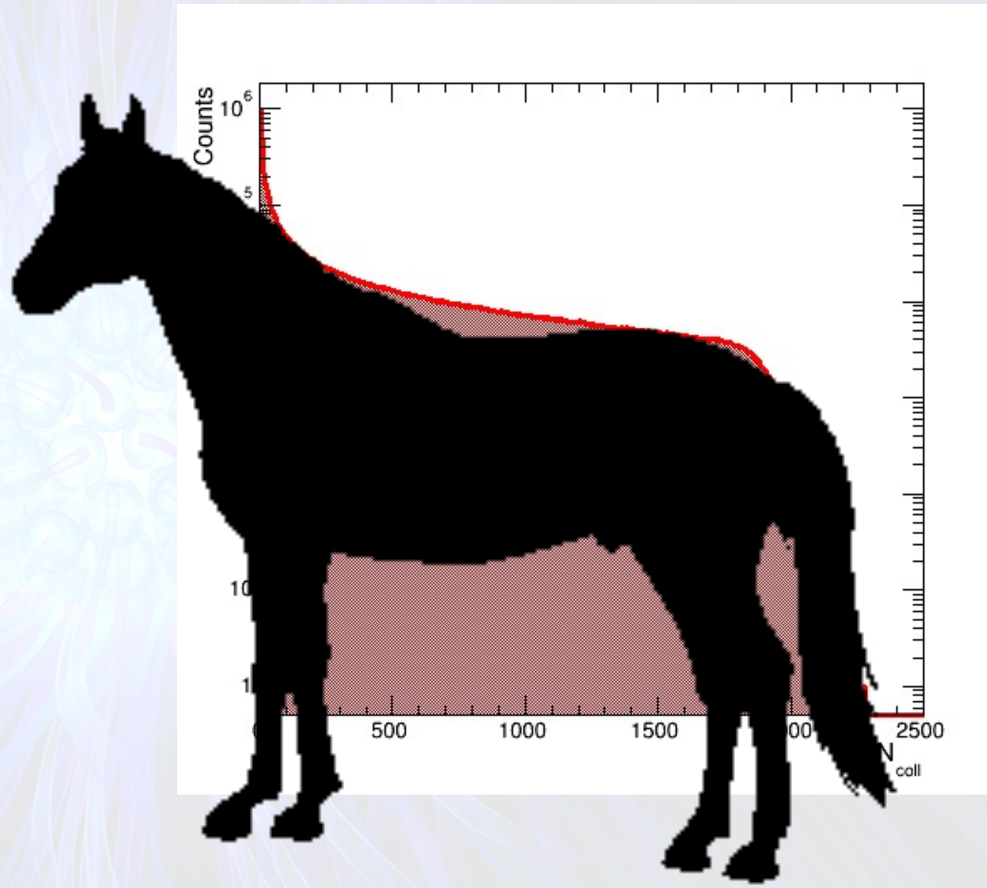
Cross Section in Nuclear Collisions

- Nuclear forces are short range
  - Range for Yukawa Potential $R \sim 1/M_x$
    - Exchanged particles are pions: $R \sim 1/(140\ \text{MeV}) \sim 1.4\ \text{fm}$
  - Nuclei interact when their edges are ~ 1fm apart
  - 0th Order: Hard sphere
  - $\sigma_{\text{geom}} = \pi (R_1 + R_2)^2 = \pi r_0^2 (A_1^{1/3} + A_2^{1/3})^2$
    - $r_0 = 1.2\ \text{fm}$
  - Bradt & Peters formula
    - $\sigma_{\text{geom}} = \pi r_0^2 (A_1^{1/3} + A_2^{1/3} - b)^2$
    - b decreases with increasing A_{min}
  - J.P. Vary's formula:
    - $\sigma_{\text{geom}} = \pi r_0^2 (A_1^{1/3} + A_2^{1/3} - b_0 (A_1^{-1/3} + A_2^{-1/3}))^2$
    - Last term: curvature effects on nuclear surfaces



Find N_{part}, N_{coll}, b distributions

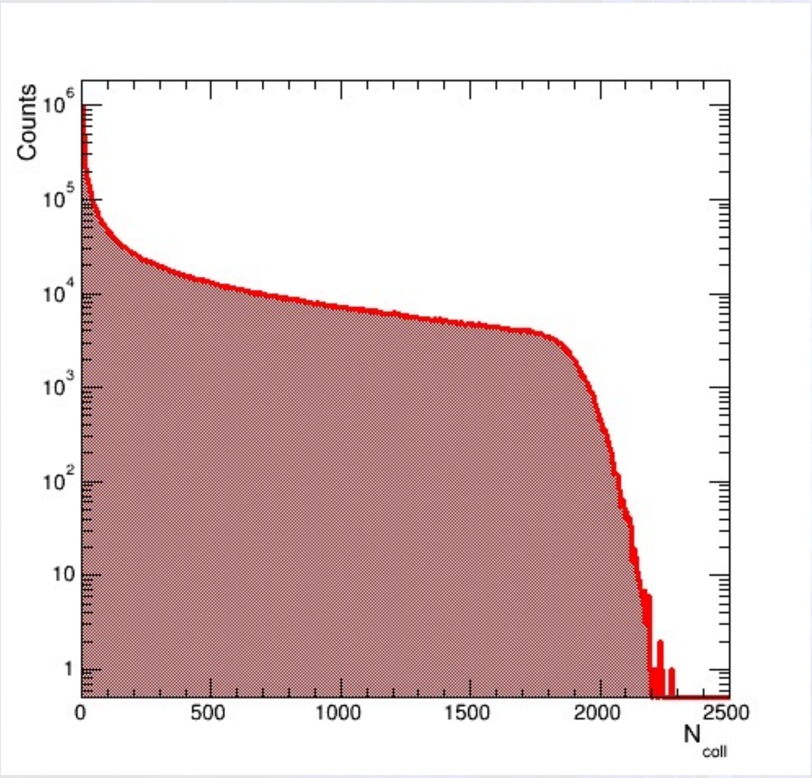


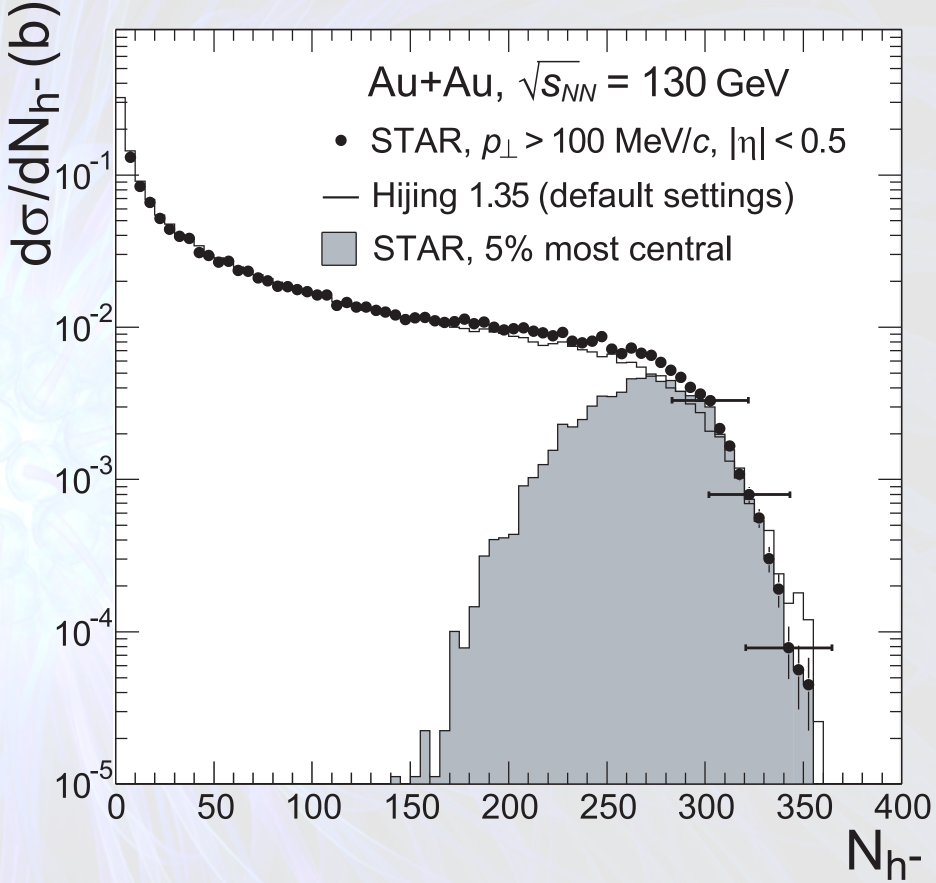


- Nuclear Collisions

From Glauber to Measurements

- Multiplicty Distributions in STAR

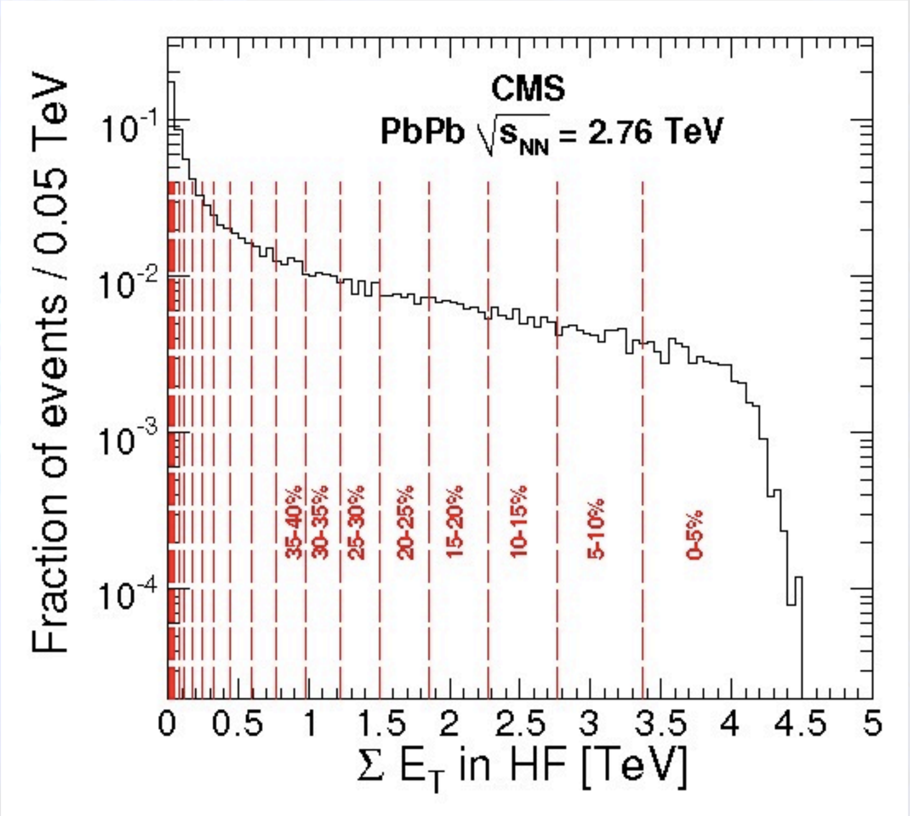




MCBS, Ph.D Thesis
Phys.Rev.Lett. 87 (2001) 112303

Comparing to Experimental data: CMS example

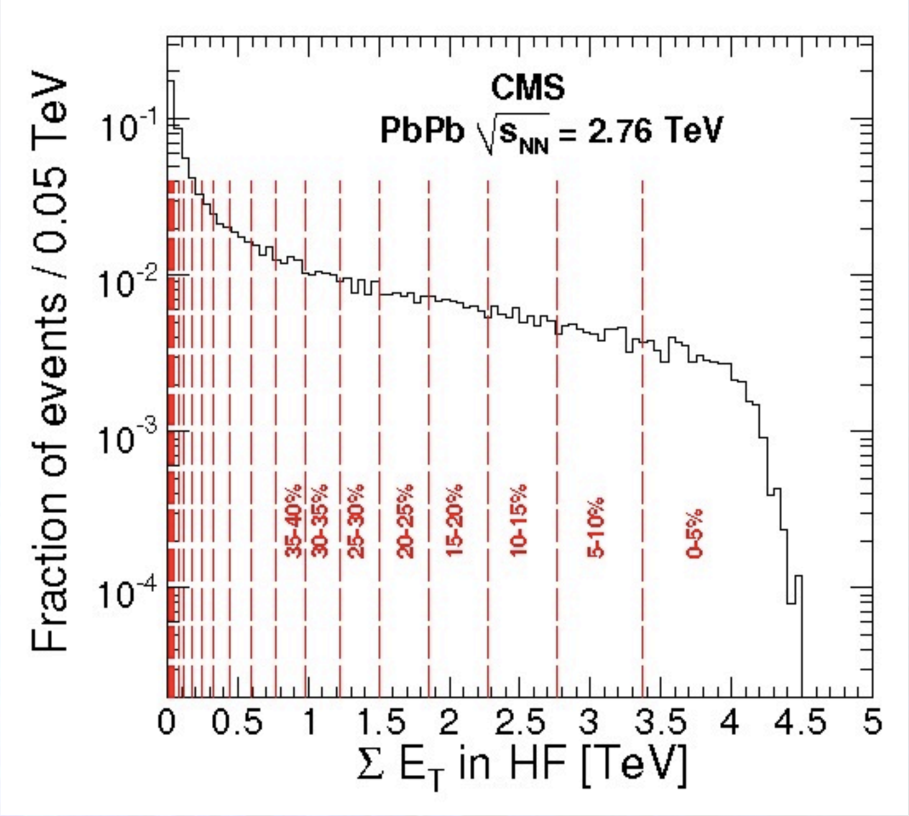
- Each nucleon-nucleon collision produces particles.
  - Particle production: negative binomial distribution.
- Particles can be measured: tracks, energy in a detector.
- CMS: Energy deposited by Hadrons in "Forward" region



Centrality Table in CMS

- From CMS MC Glauber model.
- CMS: HIN-10-001,
- JHEP 08 (2011) 141

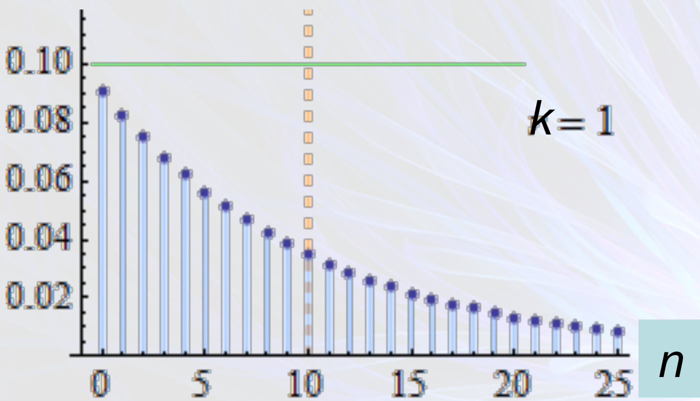
- Phobos version of Glauber MC:
  - SoftwareX 1-2 (2015) 13-18
  - arXiv:1408.2549



Centrality	0-5%	5-10%	10-15%	15-20%	20-25%	25-30%
N_{part}	381 ± 2	329 ± 3	283 ± 3	240 ± 3	203 ± 3	171 ± 3
Centrality	30-35%	35-40%	40-45%	45-50%	50-55%	55-60%
N_{part}	142 ± 3	117 ± 3	95.8 ± 3.0	76.8 ± 2.7	60.4 ± 2.7	46.7 ± 2.3
Centrality	60-65%	65-70%	70-75%	75-80%	80-85%	85-90%
N_{part}	35.3 ± 2.0	25.8 ± 1.6	18.5 ± 1.2	12.8 ± 0.9	8.64 ± 0.56	5.71 ± 0.24

UA5 Multiplicity Distributions in $p+\overline{p}$

- Fit to negative binomial:



- NBD

$$\binom{n+k-1}{n} p^n (1-p)^k, \text{ Mean: } \mu = \frac{pk}{1-p}$$

Rewrite in terms of mean μ and k :

$$(1-p)\mu = pk; \mu / k = p/(1-p)$$

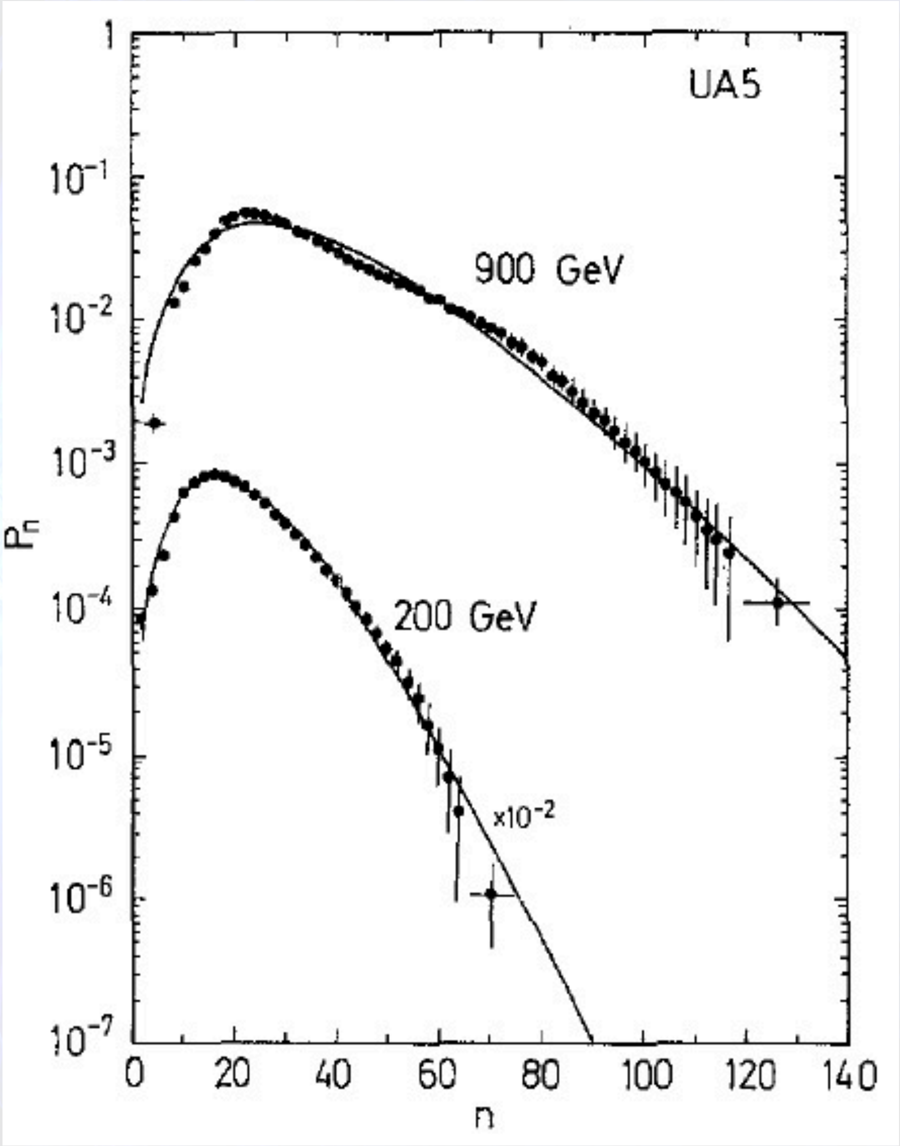
$$(\mu / k)^n = p^n / (1-p)^n$$

$$(\mu / k + 1) = (p + 1 - p)/(1-p) = 1/(1-p)$$

$$(\mu / k + 1)^{-(n+k)} = (1-p)^{n+k}$$

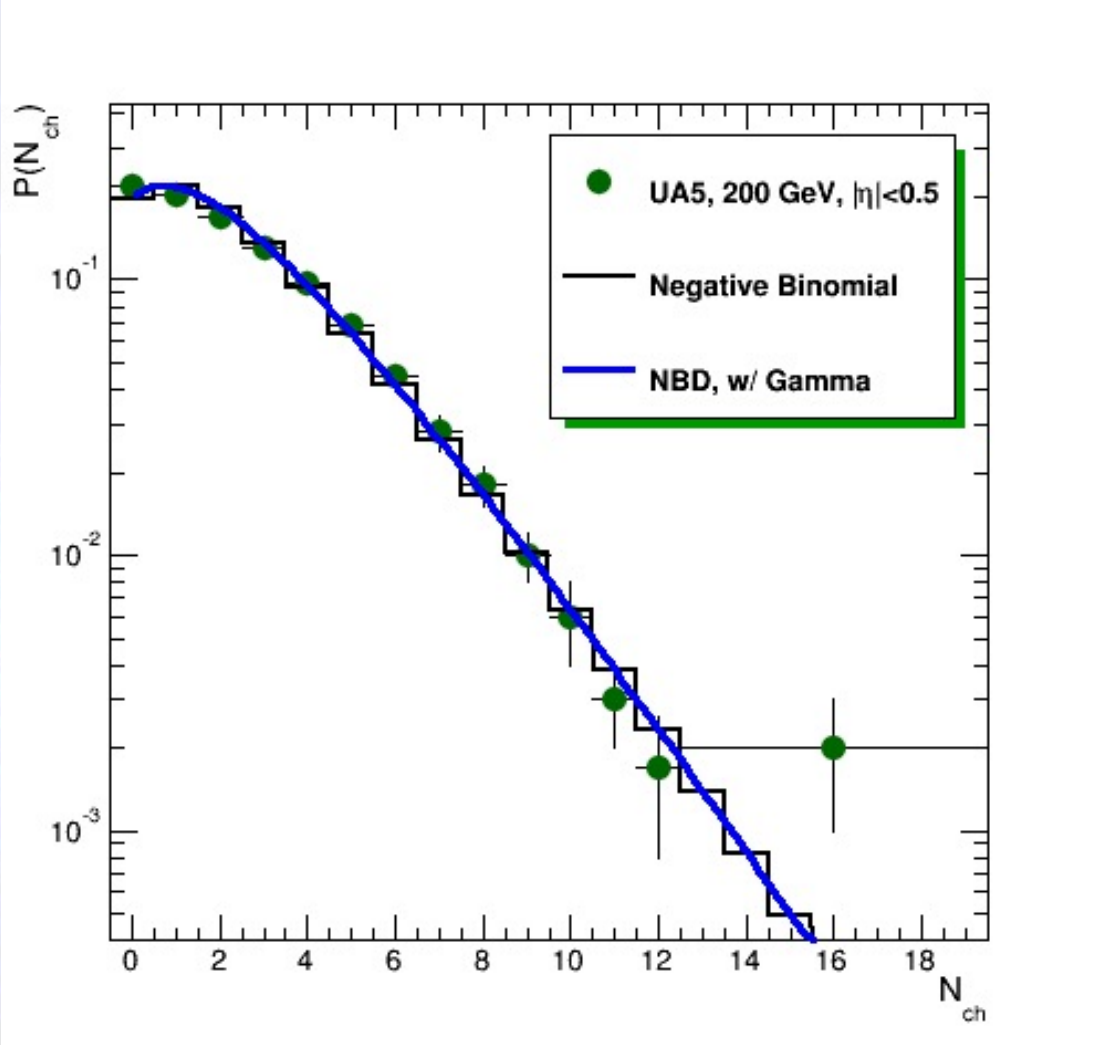
$$\frac{(\mu / k)^n}{(\mu / k + 1)^{n+k}} = p^n (1-p)^k$$

$$\binom{n+k-1}{n} \frac{(\mu / k)^n}{(\mu / k + 1)^{n+k}}$$



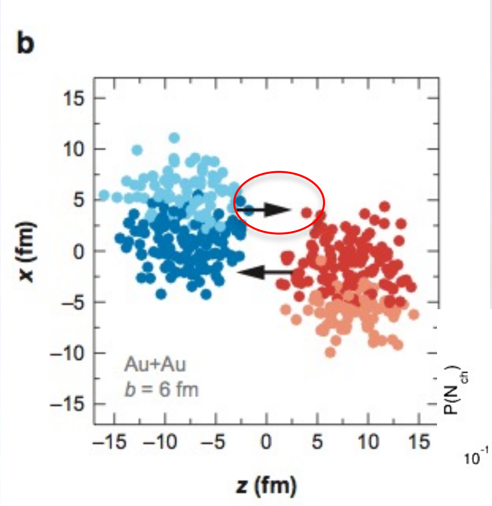
Fit of UA5 Data to NBD

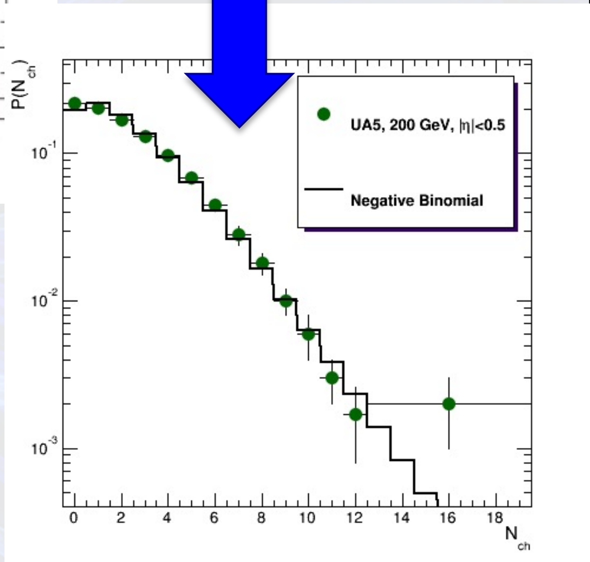
- Midrapidity data
- Fit to NBD:
  - $\mu = 2.53 \pm 0.06$
  - $k = 2.00 \pm 0.03$
- Easy to do in ROOT with Minuit
  - Data are fit to an analytic function
    - Can use Gamma functions to get a smooth curve



Fitting the E_T Distribution

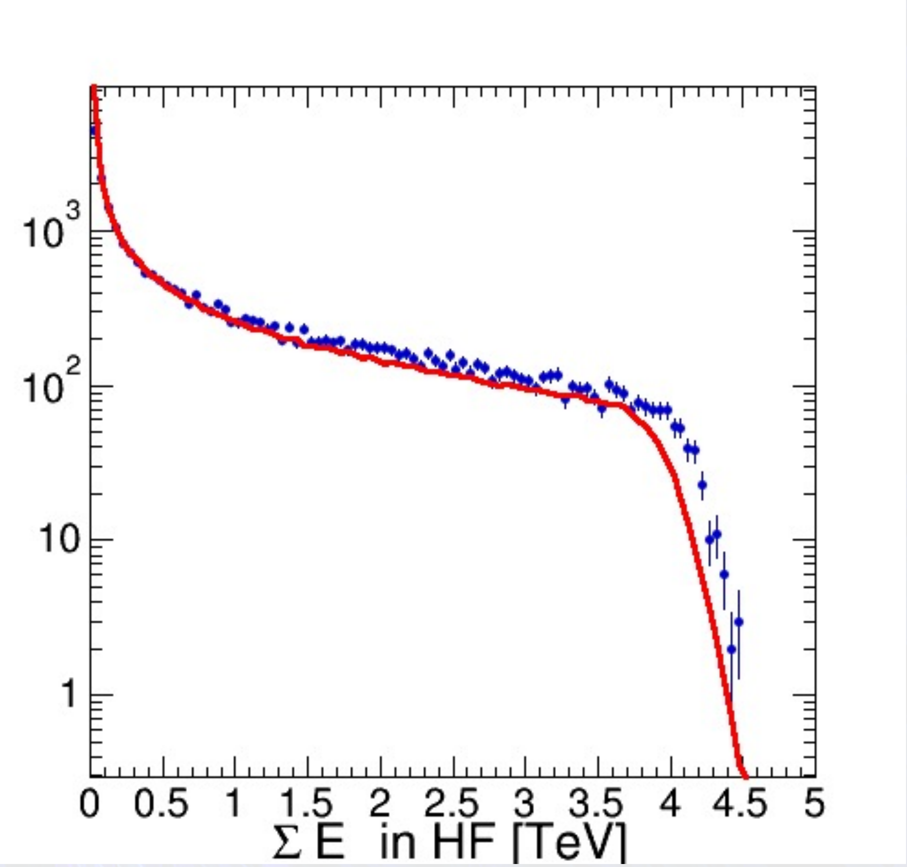
- Each binary collision:
  - Produces particles or energy
  - For a given PbPb collision, with Ncoll binary collisions:
  - Sample from NBD, Ncoll times.
  - Gives one realization (pseudo-event) for that value of Ncoll
- Obtain a distribution for that value of Ncoll by repeating the above many times.
- Sum the results for all values of Ncoll
- This is no longer analytic!





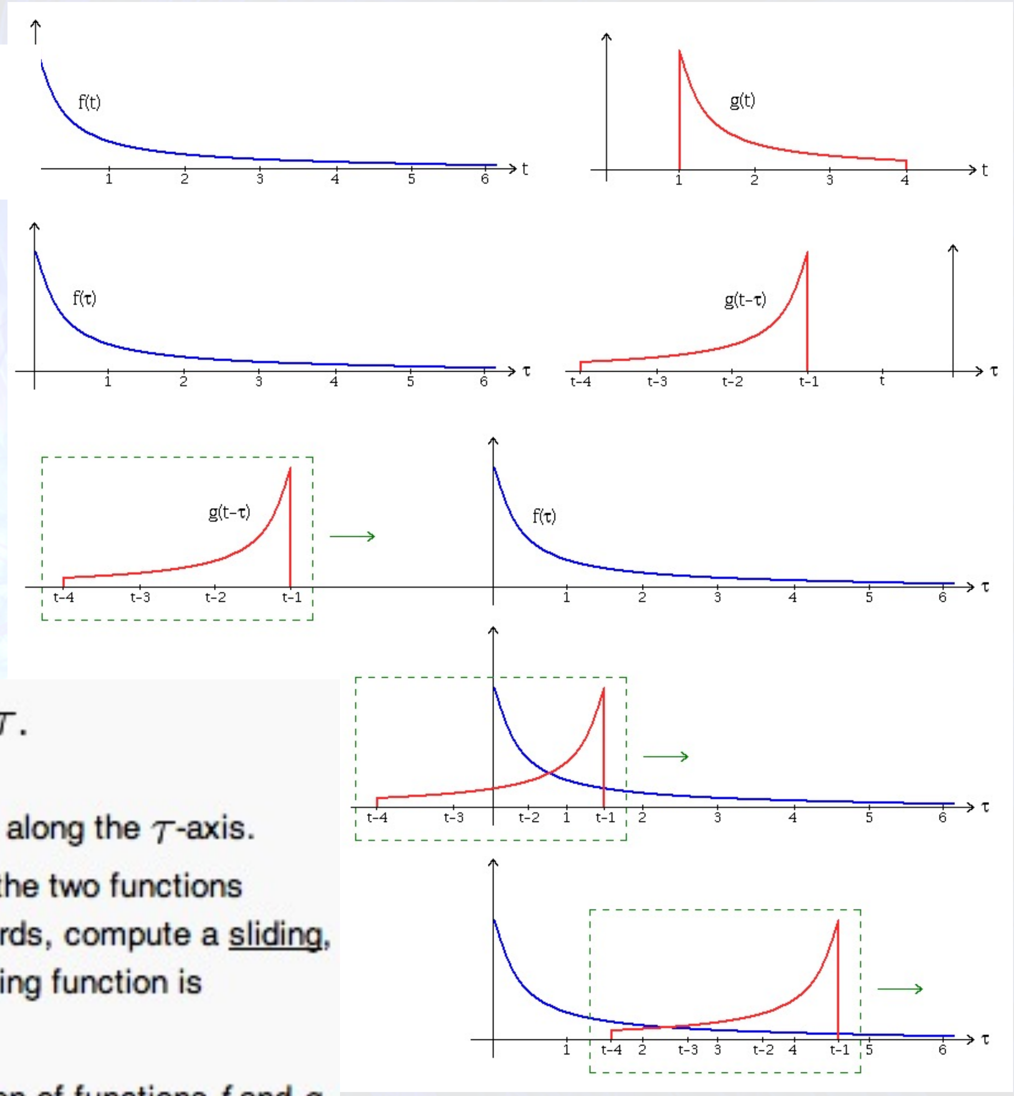
Resulting distribution

- Shape can be similar to experiment
  - Will depend on choice of parameters: k, <n>
    - Here:
      - k=0.96
      - <n>=1.6
- In general, won't match data exactly.
  - Need to **fit** the parameters.
- Note: fit from 0.5 to 5 TeV to avoid inefficiency from peripheral collisions



Convolution

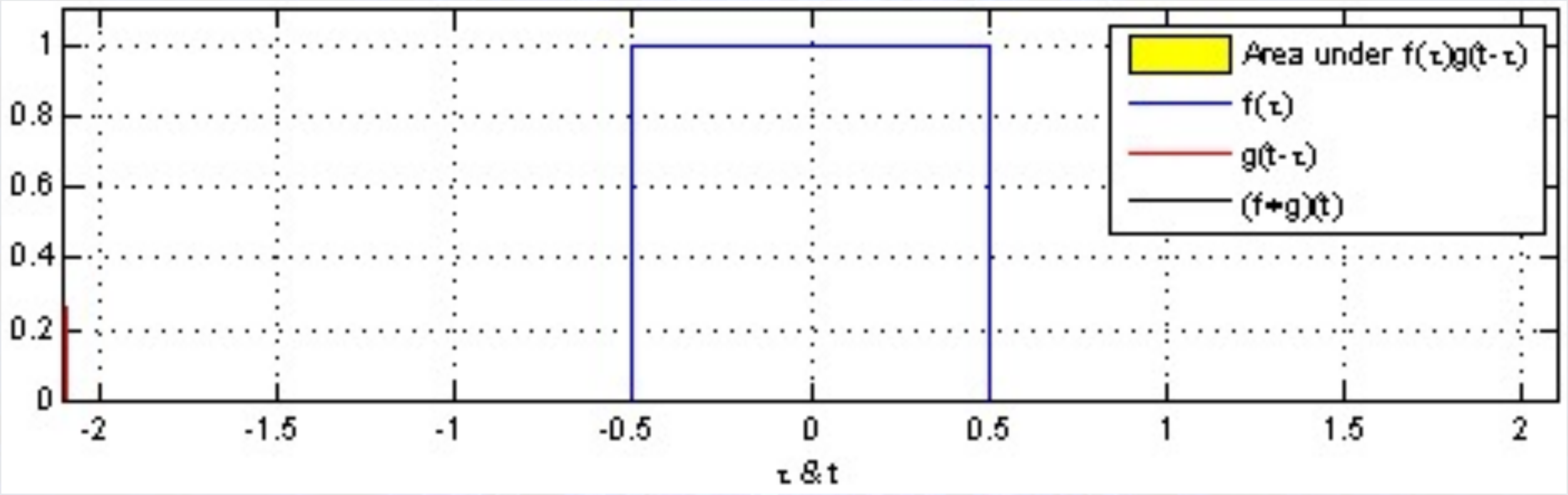
$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$



1. Express each function in terms of a dummy variable τ.
2. Reflect one of the functions: $g(\tau) \rightarrow g(-\tau)$.
3. Add a time-offset, t, which allows $g(t-\tau)$ to slide along the τ-axis.
4. Start t at $-\infty$ and slide it all the way to $+\infty$. Wherever the two functions intersect, find the integral of their product. In other words, compute a sliding, weighted-average of function $f(\tau)$, where the weighting function is $g(-\tau)$.

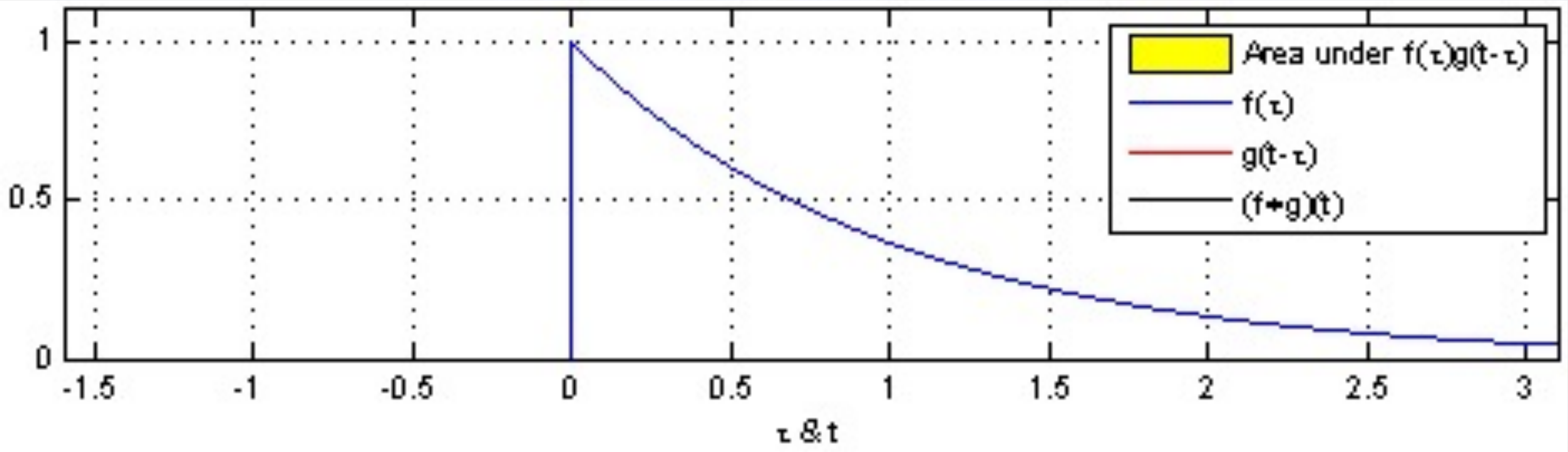
The resulting waveform (not shown here) is the convolution of functions *f* and *g*.

Convolution



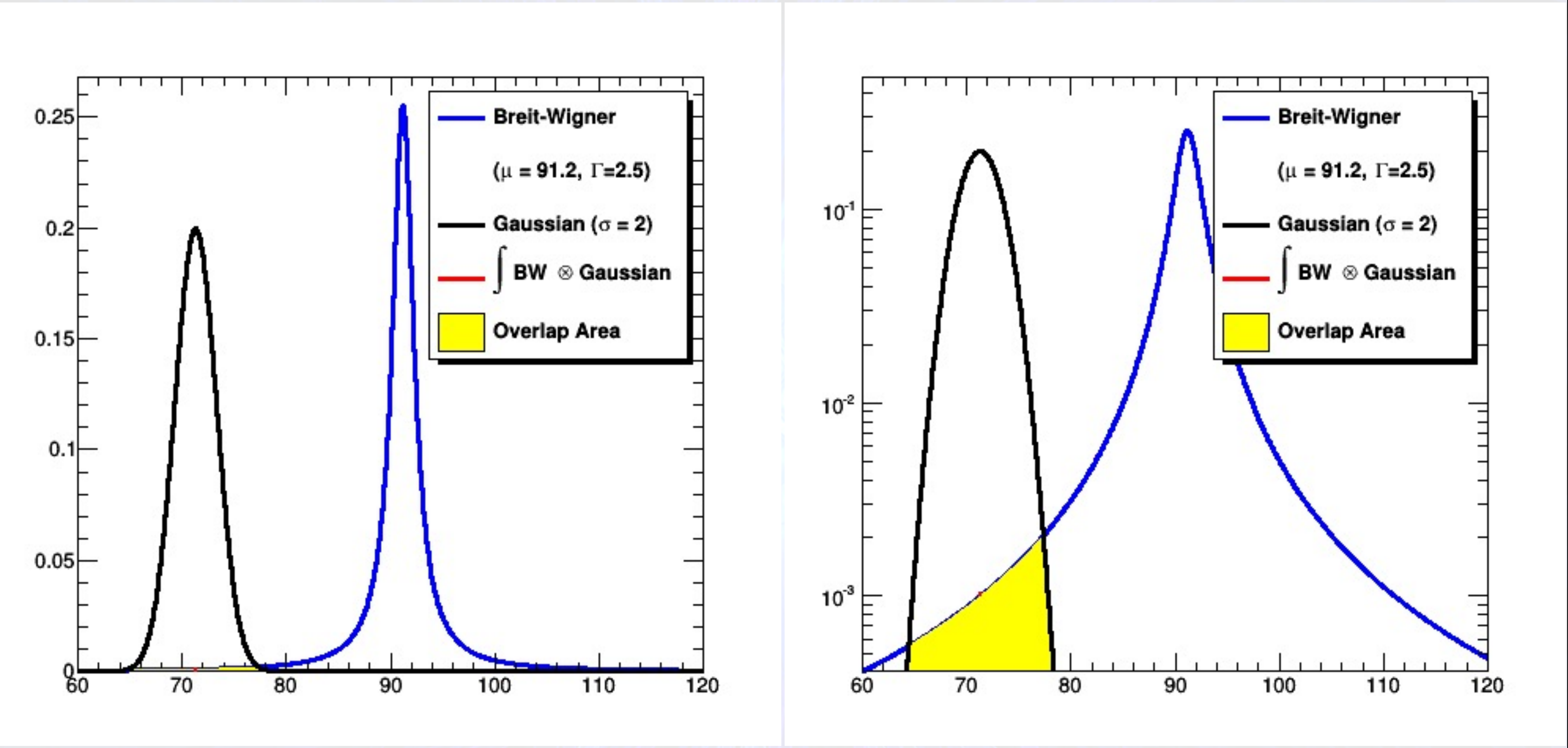
- As an animation

Convolution



- Of a box and a truncated exponential-like function.

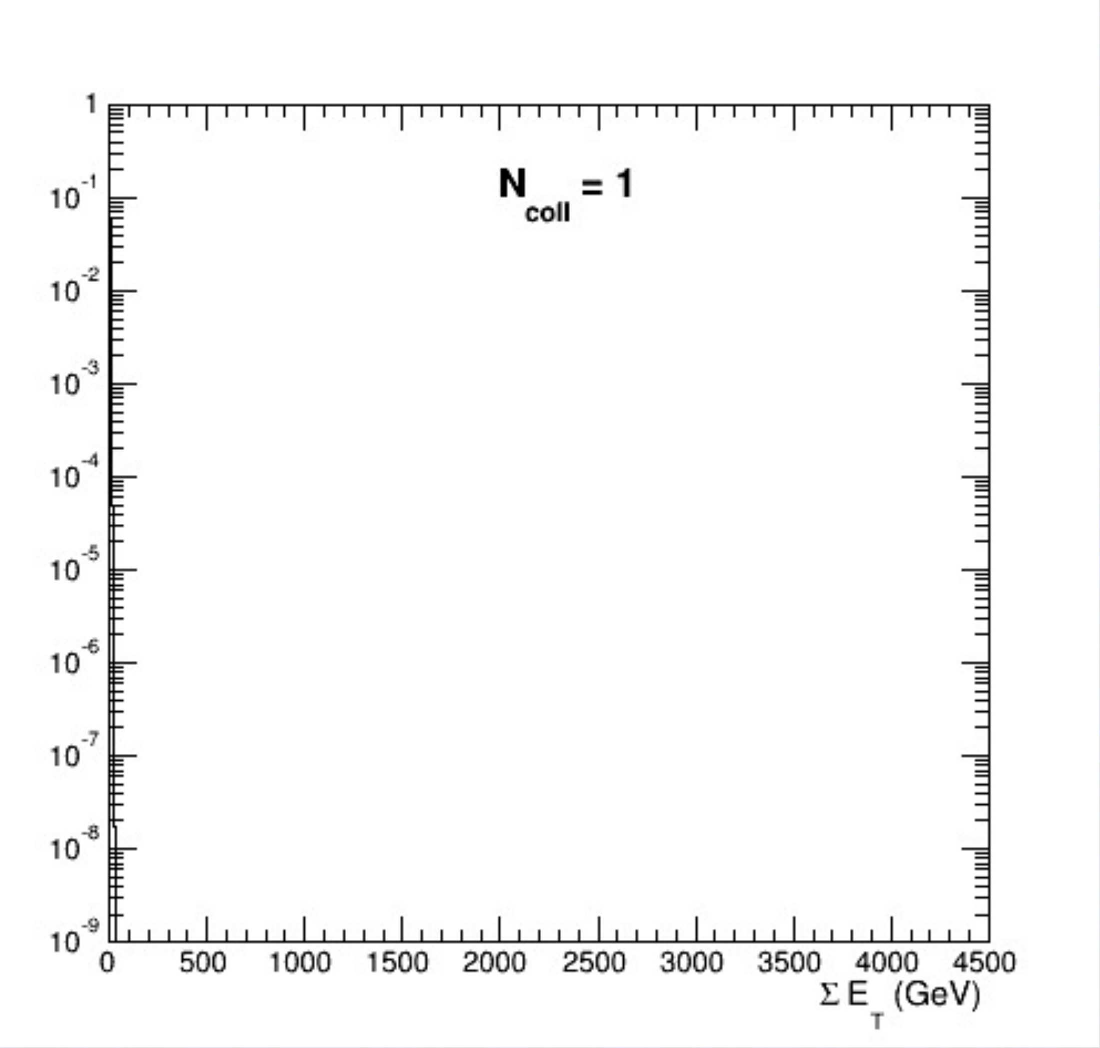
Convolution of Breit-Wigner & Gauss



- Animation code by Chris Flores

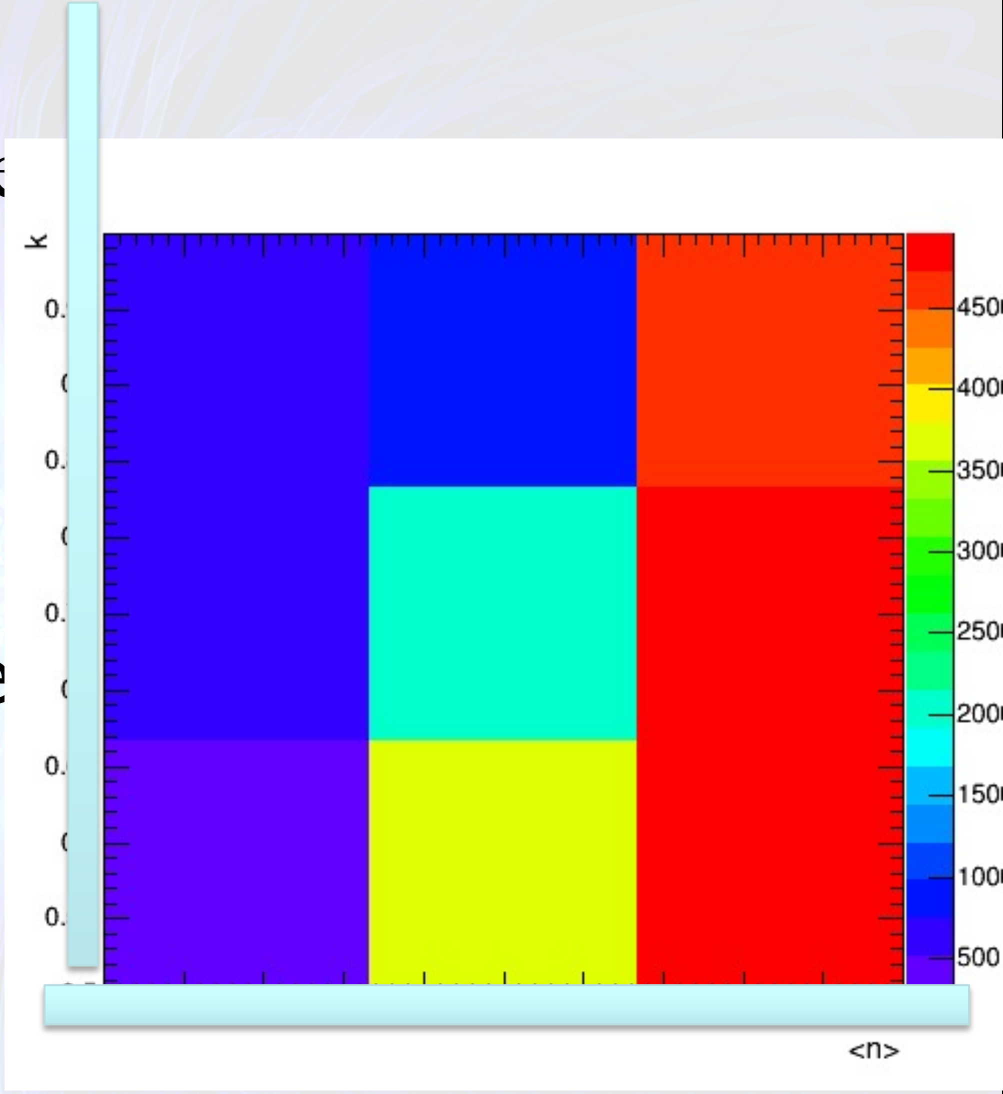
Convolution Animation, Drawing from NBD N_{coll} times.

- Convolving N_{coll} with Negative Binomial.



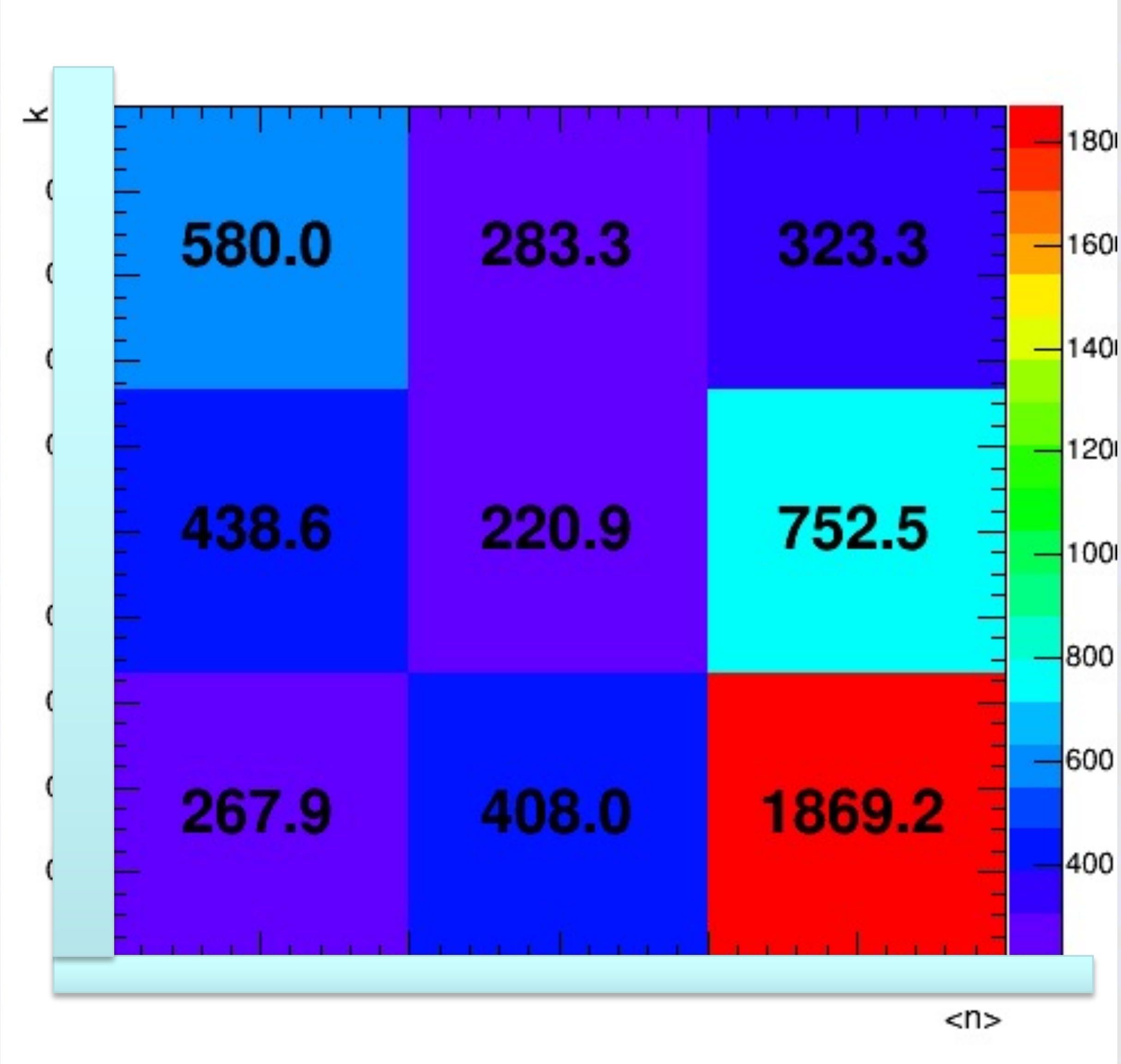
Scanning the parameter space

- For the Glauber MC, scan the parameter space by hand
  - First do a coarse pass
    - k = [0.5,1.5]
    - <n>=[0.5,1.5]
  - Then, zero in on range
  - 2nd pass, plot:
    - Minimum χ^2~500
    - If minimum is not centered, need to re-center the range
    - Here, minimum in corner



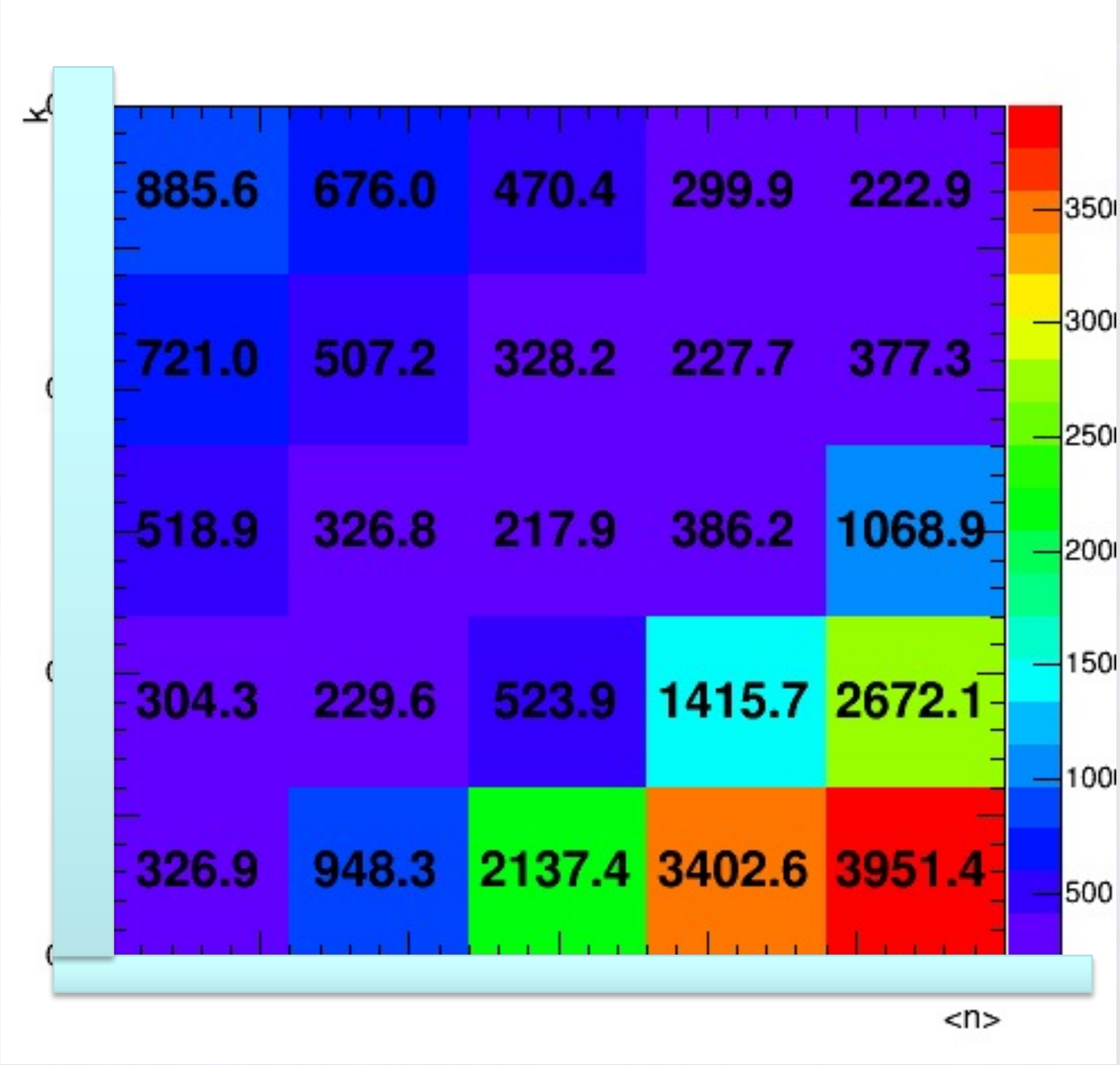
...and a 3d pass

- Minimum near center of range



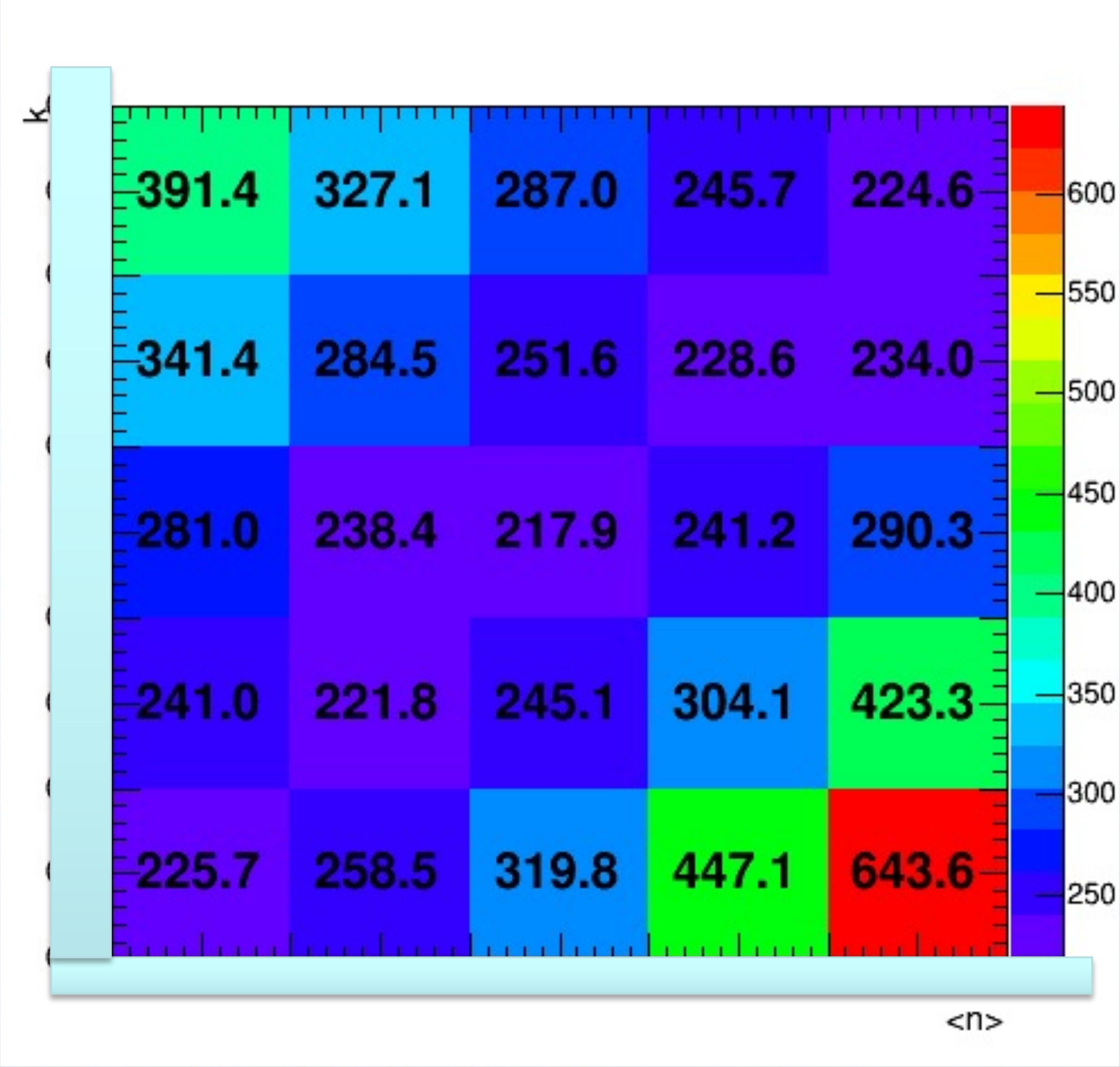
...and a 4th pass

- Minimum near center again.



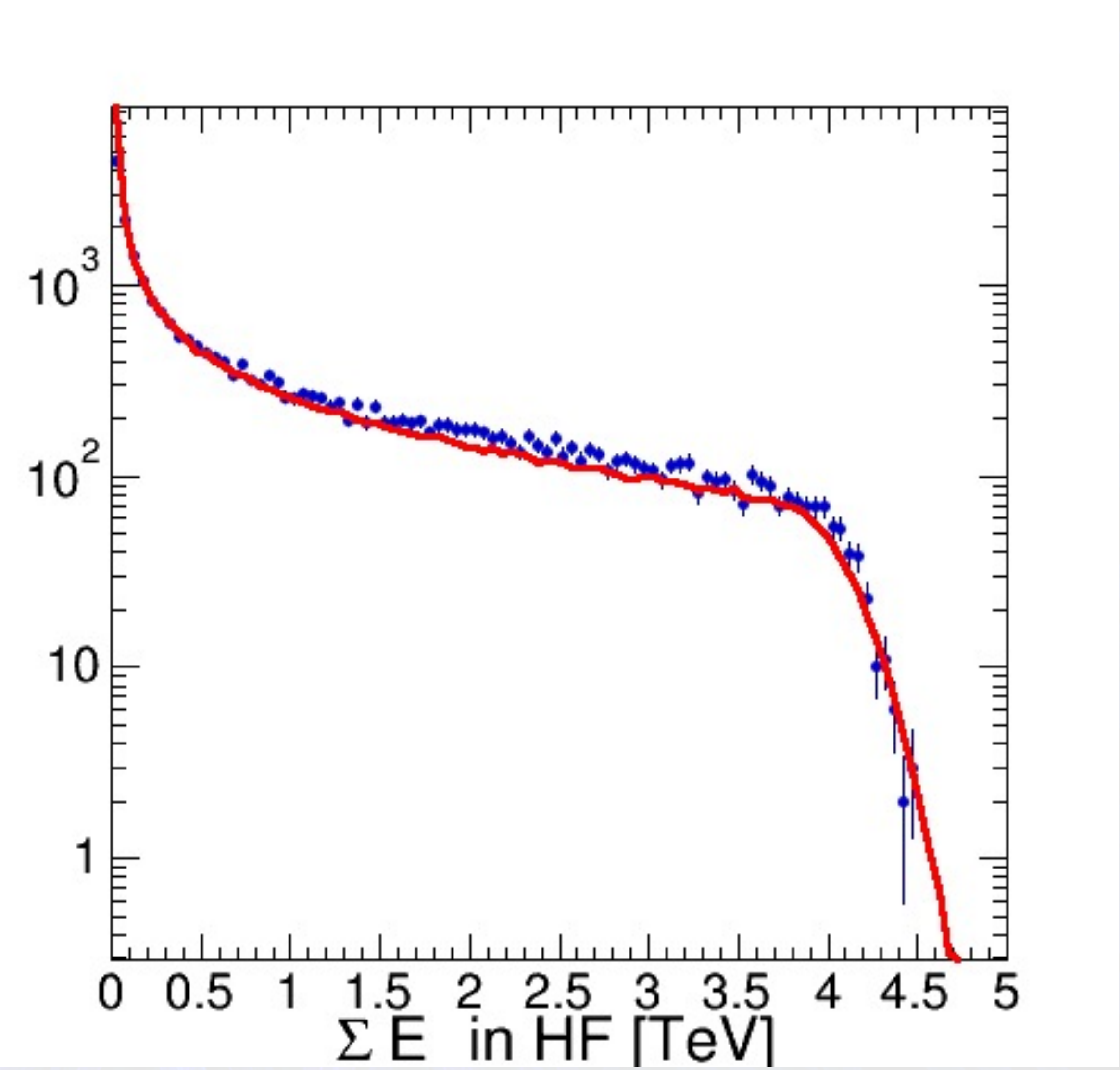
...and a 5th pass

- Minimum didn't change after zoom.
- Note correlation between the parameters!
- Have not reached level of $\Delta\chi^2$=1
  - Since function will have fluctuations, might not reach it



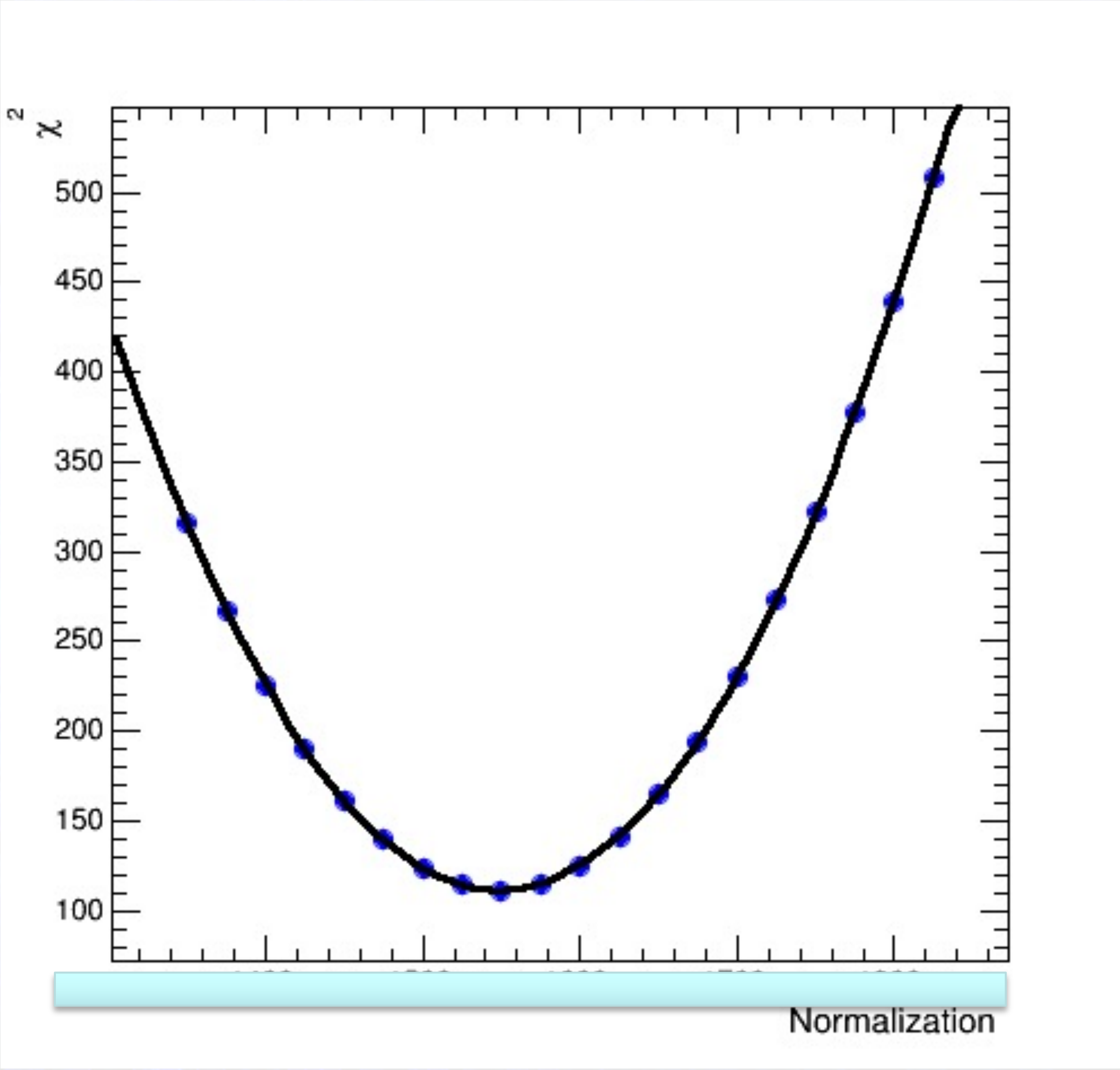
With those parameters...

- Use parameters k, <n> found in previous step.
- Normalization is not yet optimal.



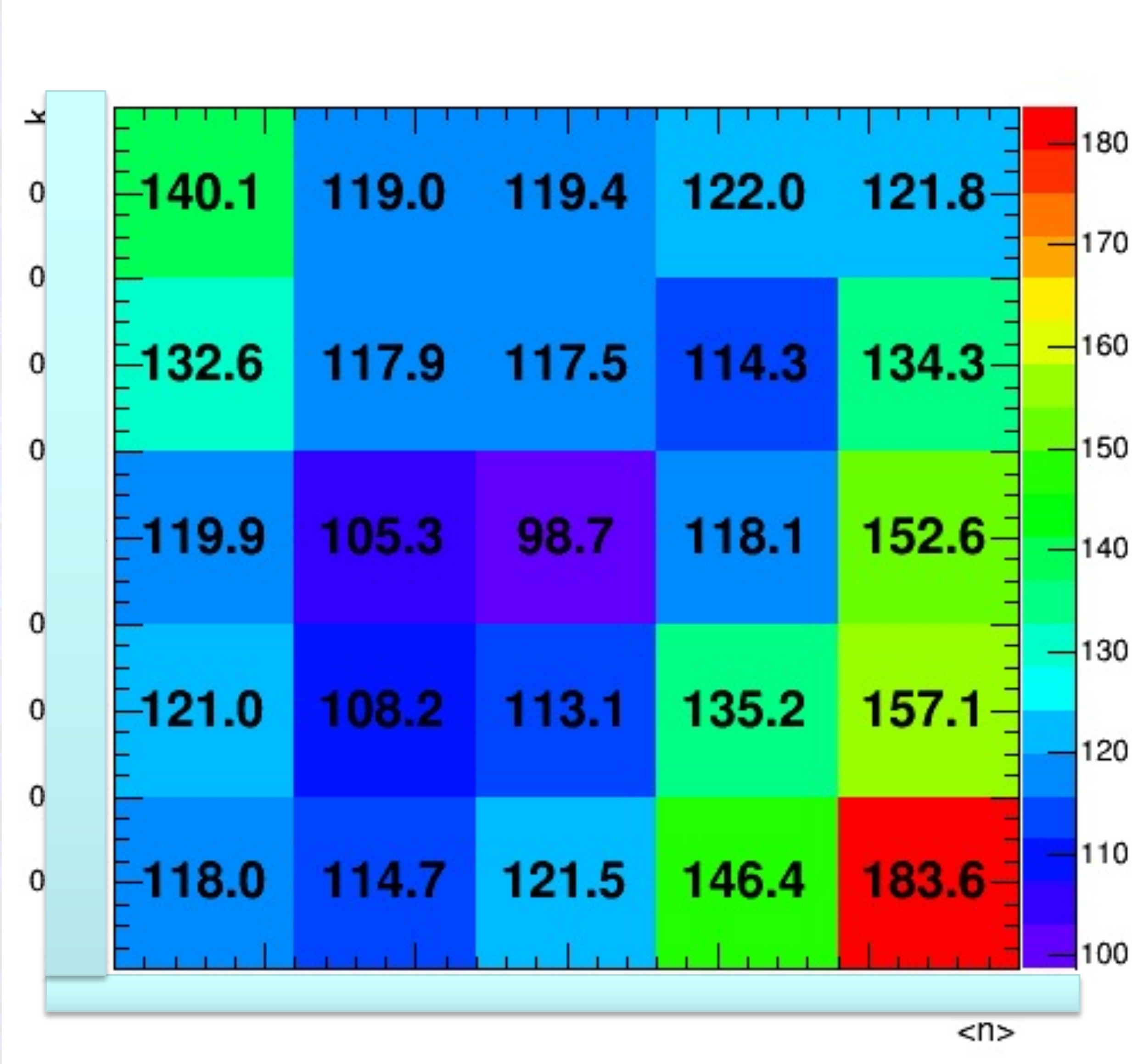
Minimizing χ^2 for Normalization...

- Vary Normalization
- Obtain value at minimum χ^2.



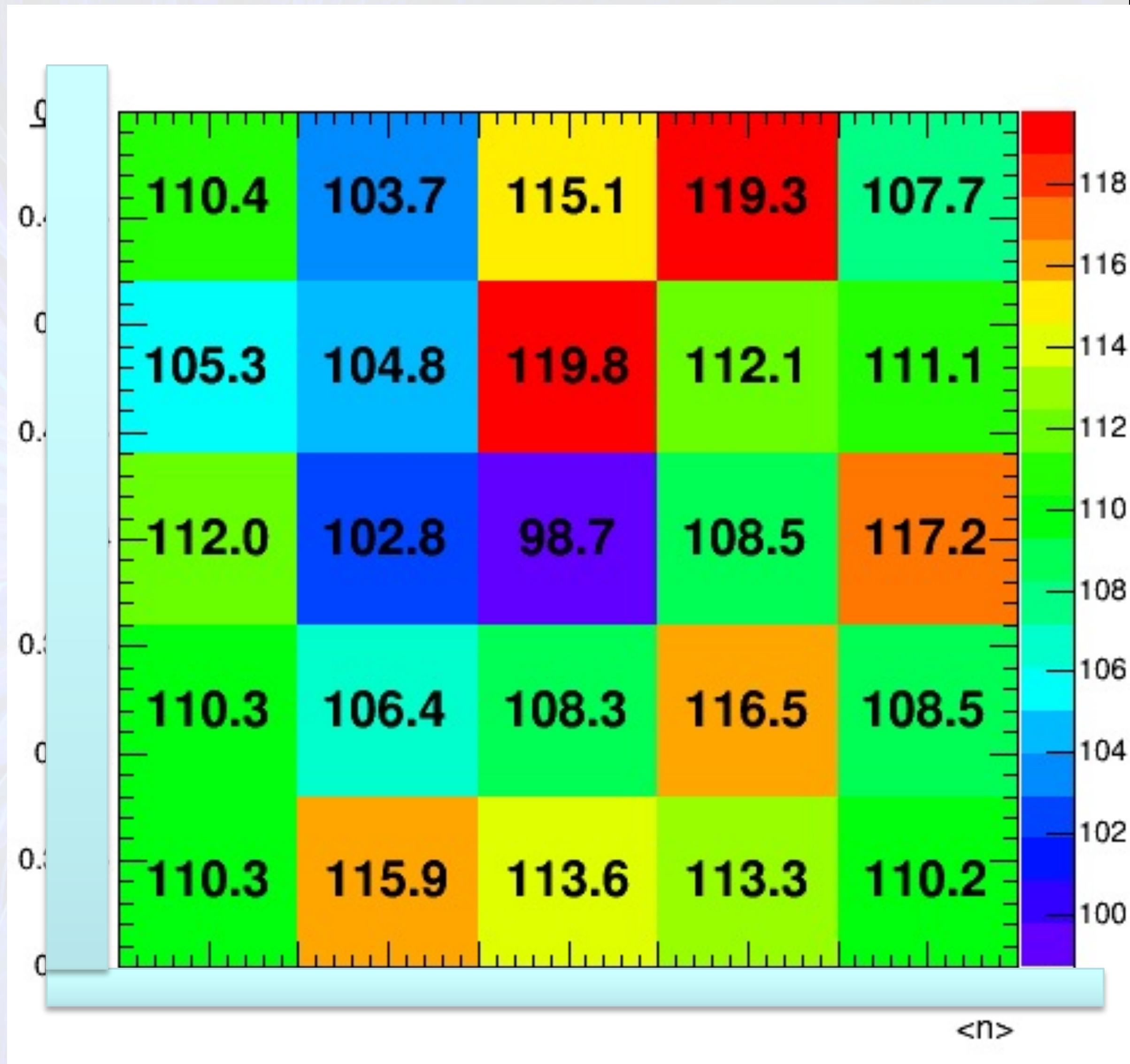
Rescan NBD parameters...

- Keep Normalization at minimum.
- Rescan k, <n>
- Minimum stayed about where it was:
  - Little correlation between Normalization and NBD parameters

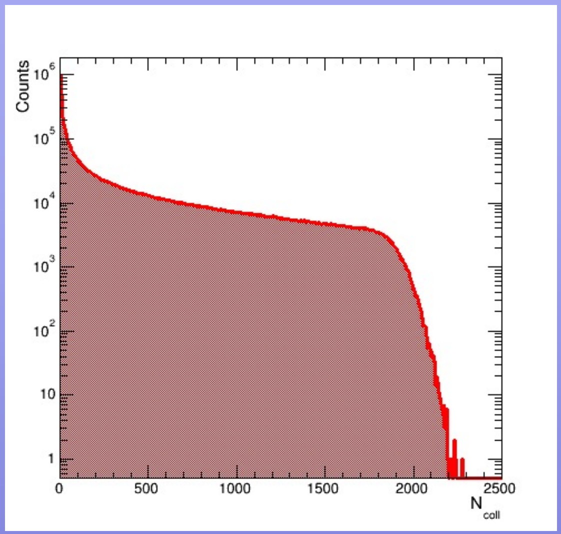


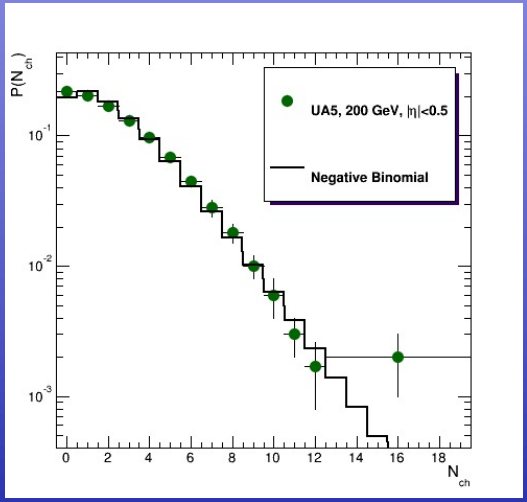
Last scan...

- Found no change
- Fluctuations start to appear
  - Changes in range:
  - $\Delta k = 0.004$
  - $\Delta \langle n \rangle = 0.01$
  - Error estimates:
    - $\sigma_k \sim 0.0005$
    - $\sigma_{<n>} \sim 0.001$



Final fit





- **Model** and **data**