

Probability Distribution Functions: Stochastic Processes in Physics

#### Stochastic variables:

- Variables that fluctuate from one realization of a system to another.
  - o Thermal effects.
  - Manufacturing uncertainties.
  - Quantum processes.

Simple example: The 1-D Random Walk.



# Random Walk and Stochastic Processes

RW: The particle will take N steps. At each step, there is a 50/50 chance for the particle to move right or left.

Physical System is characterized by parameters that vary randomly.

- RW: two discrete values: ±1 distance units
  - + is right, is left
- Want to calculate a global parameter that can be evaluated or measured.

• RW: Total initial displacement from the origin after N steps.

- General problem: Predict the probability that the global variables possess a specific value when averaged over all trial experiments.
  - RW: Probability that the walk terminates at a given displacement from the origin.

### Making a random walk program in root:

int numberOfSteps = 40; int numberOfRealizations = 1e4;

TH1D\* mRandomWalkHisto = new TH1D ("mRandomWalkHisto","Random Walk",2\*numberOfSteps+1,-numberOfSteps-0.5,numberOfSteps+0.5)

# Code for loop

TRandom3 rnd3(0); // initialize random number generator with unique seed for (int iRealization = 0;

iRealization<numberOfRealizations;

++iRealization) {

int position = 0;

for (int iStep = 0; iStep<numberOfSteps; ++iStep) {</pre>

double a = rnd3.Rndm(); //random number between 0-1

double step = 1;

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if (a<0.5) step=-1;
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// at this point, 50% of the time step will be 1

// and 50% of the time step will be -1

position += step;

} // loop over steps

//cout << "Realization " << iRealization << ", position " << position <<
endl;</pre>

mRandomWalkHisto.Fill(position);

}// loop over realizations

## **Result: Random Walk Histogram**



# Examples of Histograms of Random Distributions



#### Homework 8, Random-Walk

Code the Random Walk program in ROOT.

Modify it to use a 2-D Histogram to do a 2-D random walk with unit length steps in which the angle that the walker describes with respect to any fixed axis is a uniformly distributed random variable on [0, 2π]. Use 40 steps, and also use unit width.